Teransportation and Assignment Problems

The transportation peroblem:

The transportation problem is to transport various amounts of a single homogeneous comodity, that one initially stored at various origins, to different distinations in such a way that the total transportation cost is minimum.

Methods to find initial feasible solution

- 1. Nonthwest conner method (NWC)
- & Matrix Minima method
- 3. Vogels Approximation method. (VAM)

Morth West Corner Method (NWC)

Step 1: Identify the northwest corner of the table

Allocate x11 = min (a,, b,)

case 1: of a, < b, , then first now gets completed.

case &: [] b. < a., then first column gets completed

case s: 1) a, = b, , then there is a tie and

allocation can be made arbitrarily.

Step 2: Start from the northwest corner and richeat step I until all the requirements are satisfied

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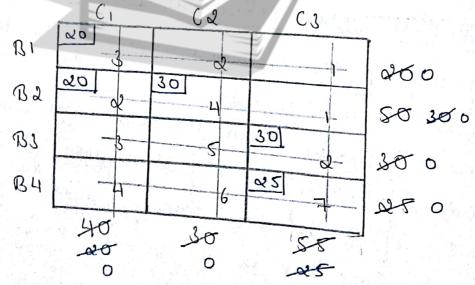
oli-find the initial fearible solution for the following transpositation problem by us using nous, west council method.

	CI	C2	C3	Jupply
BI	3	2		80
B2	ď	etti cominimi punti portuga prima pr		50
B3	3		2	30
B4	Long	6	7	025
	40	30	55	

Step 1: Supply = 00+50+30+25 = 125 Wemand = 40+30+55=125

Demand

Supply = Demand, Hence the given transportation purblim is balanced.



dtep & The NWC is (1,1), x11 = min (20,40)

DO is allocated to xxx (1,1)

step 3. The NWC = (0,1) x21 = min (20,50) = 20 20 is allocated to (2,1), (1 completes.

step 4. The NWC is (2,2) N22= min (30,30)=30 30 is allocated to (2,2) C2 and B2 are complete.

30 is allocated to (3,3) B3 is complete.

step 6: The NWC is (4.3) N43 min (25,25) = 25 25 is allocated to (4.3) cs and B4 asee complete.

The total cost TC=(20x3) + (20x2) + (30x4) + (30x2) + (25x7) = 455

pa: Find initial feasible solution by applying nonthwest council method:

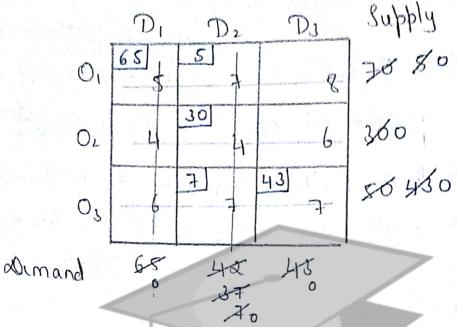
		Di	D2	$\mathcal{P}_{\mathfrak{I}}$	Supply
	01	5	ユ	8	70
	೦೩	4	4.	6	30
	03	6	7	-	20
Dimo	ind .	65	42	43	

8461: 8464y= 70+30+50=150

Demand = 65+42+43=150

Supply = Demand, Hence given transpositation

Decoblim is balanced.



Step 8: The NWC is (1,1), $x_{11} = min(70, 65) = 15$ 65 is allocated to (1,1), D1 is complete

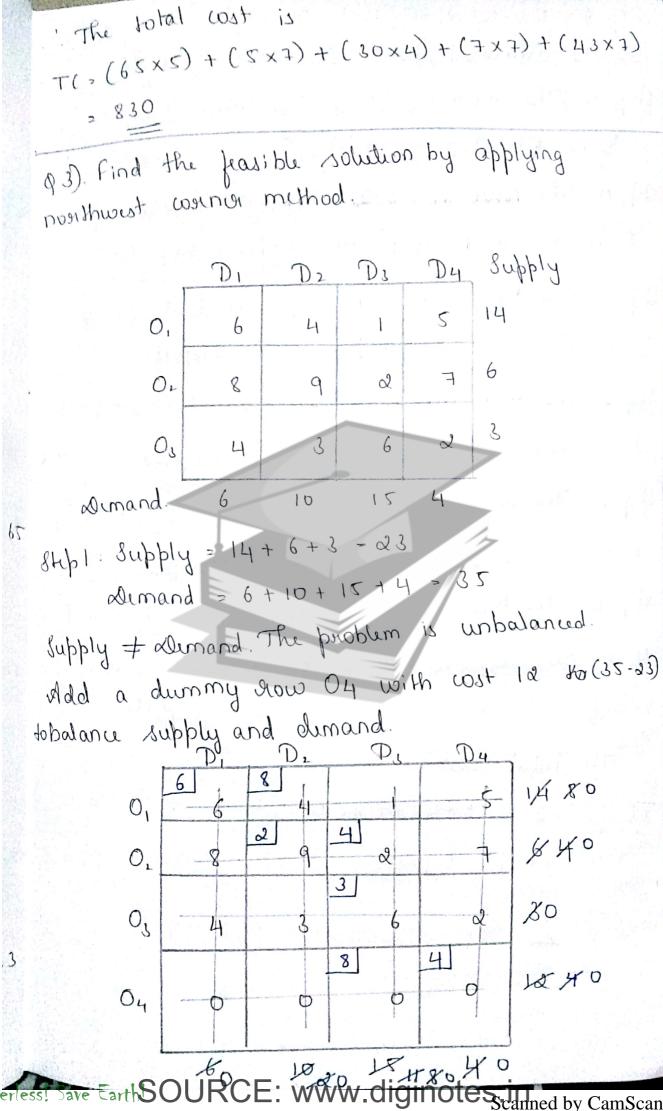
5tep 3: The NWC is (1,2) $x_{12} = min(5, 42) = 5$ 5 is allocated to (1,2), D_1 is complete

5tep 4: The NWC is (2,2) $x_{22} = min(30,37) = 30$ 80 is allocated to (2,2) O_2 is complete

5tep 6: The NWC is (3,2) $x_{32} = min(50,7) = 7$ 7 is allocated to (3,2), D_2 is complete

5tep 6: The NWC is (3,3) $x_{33} = min(43,43) = 43$ 43 is allocated to (3,3) O_3 and O_3 are

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Sty 2: The NWC is (1,1), x11= min (14,6) = 6 6 is allocated to (1,1), D, is complete Step 3: The NWC 12 (1,0) x12=min (8,10)=8 8 is allocated to (1,2) O, is complete Shp 4: The NWC is (0,0) x22 min (0,6) = 2 d is allocated to (2,2) D2 is complete Step T. The NWC is (0,3) 723 = min(15,4) =4 4 is allocated to (2,3) O2 is complete Step 6: The NWC is (3,3) x332 min (3,11)=3 3 is allocated to (3,1) Os is complete ship 7: The NWC is (4.3) 2432 min (8,12),8 8 is allocated to (4,3) Do is complete step 8: The NWC is (4,4) N44 = min (4,4) = 4 4 is allocated to (4,4) & 04 and D4 are complete. , The total cost is T(= (6x6) + (8x4) + (2x9) + (4x2) + (6x3) $+ (8 \times 0) + (4 \times 0)$ = 112

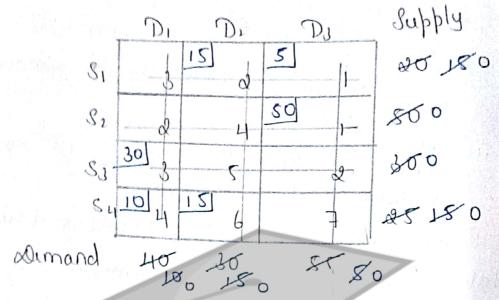
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Least cost method (Matrix Minima Method) Ship! Determine the smallest cost in the transporta - Hon table det it be (ij . Allocate = min (ai, bj) skipa: i) I Nij = ai, then cross out ith now goto step 3. ii) If xij = bj, then cross out jth coloumn. Goto step 3 iii) Il xij = ai = bi, then crow out ith now on ith voloumn, but not both step 3: Repeat steps I and 2 for rusulting transporttation table until all suguiruments are skip 4: Whenever minimum cost is not unique, make an arbitary choice among the minima. Q1>

	D	\mathbb{D}_2	D3	Supply
\$1	3	2		20
\$2	&	4	١	50
\mathcal{S}_{J}	3	5	2	30
84	4	6	7	Q T
Dimand	40	30	55	0.744

8thp 1: Supply = 20+50+30+25 = 125 Demand = 40+30+55 = 125

Decoblem is balanced.



Step &: The host wost is 1, there is a tie between (1,3) and (0,3). Find out the well to which maximum cost can be allocated i.e (2,3) = 50, S& is completed.

8hp 3: The heast cost is 1. Allo gata/min min; = (5,00)=5 Allocate 5 to (1,3) D3 is completed.

8th p 4: The least cost is & . xy x12 = min(30,15)=13 Allocate 15 to (1,0)

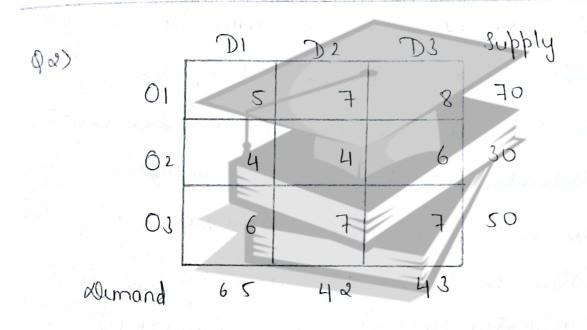
Il is completed.

84p 5: The least cost is 3. 231=min(30,40):30 dllocate 30to (3,1), S3 is complete

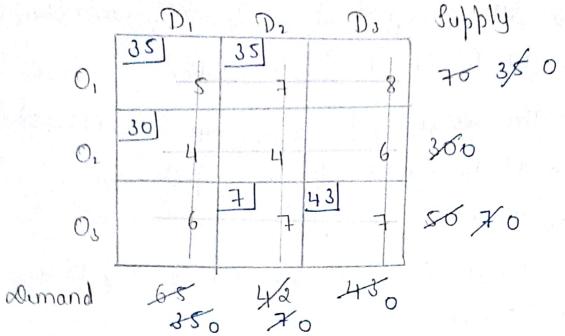
step 6: The least cost is 4 x41 = min (10,05)=10 Allocate 10 to (4,1) D1 is complete.

stip 7. The least cost is 6 x42= min (15,15)=15 idlocate 15 to (4.2) Dz is complete.

 $T(-(15 \times 8) + (5 \times 1) + (50 \times 1) + (30 \times 3) + (10 \times 4)$ $+ (15 \times 6)$ = 305



Dernand = Supply. Hence the given transportation Peroblem is bolanced.



Stip 2: The least cost is 4. maximum cost can be allocated to (2,1) N12min (30,65) = 30 30 is allocated to (2,1) 0, is completed. 8 tep 3: The least tost is 5. ni = min (70,35) = 35 35 is allocated to (1,1) D, is completed. Step 4: The least cost is 7, maximum cost can be allocated to (3,3) 23,2 min (50,43) = 43 43 is allocated to (3.3) Do is completed. step 5. The least wort is 7. maximum cost can be allocated to (1,2) N12 = min (35,42) = 35 Oi is completed Step 6: The heast cost is 7 2525(7/7) N32= min (7,7)=7 7 is allocated to (3,2)

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D. and O, are completed.

The lotal cost is:

TC = (35×5) + (35×7) + (30×4) + (7×7) +

(43×7)

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Vogel's appearing ation Method (VAM)

I find mitial feasible solution by applying VAM
method.

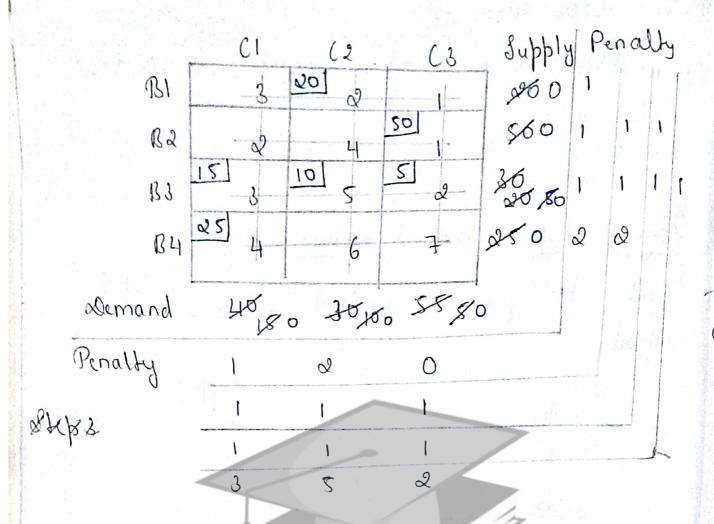
	Cı	Cz	Cs	Supply
BI	3	2	And the second control of the second control	20
R.	Q	L		50
$\mathbb{B}_{\mathfrak{J}}$	ડ	5	2	30
R4	LA	6	7	25
Demand	40	30	55	

=) Supply = 20+50 +30+25 = 125 Semand = 40 + 30 + 55 = 125

Supply = Demand. Hence the given transpositation problem is balanced.

this Add a penalty column. Find & least all in the now and find the difference. The susult is added to the penalty whoums.

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Ships find the maximum penalty a in both show and coloumn. Here the maximum penalty is a for both B4 and C2 find the least cell in B4 and C2 and assign the cost. Here do is assigned to (1,2), B1 is completed.

Ship 4 (alculate new penalty for the remaining shows and column. Paperat step at to Repeat ships at to 4 until all the rows and column are completed.

P

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a

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8th S: Calculate the total cost for all the allocated steps allocated steps

Total cost:

TC = (20x2) + (50x1) + (15x3) + (10x7) + (5x2)

+ (25x4)

= 295

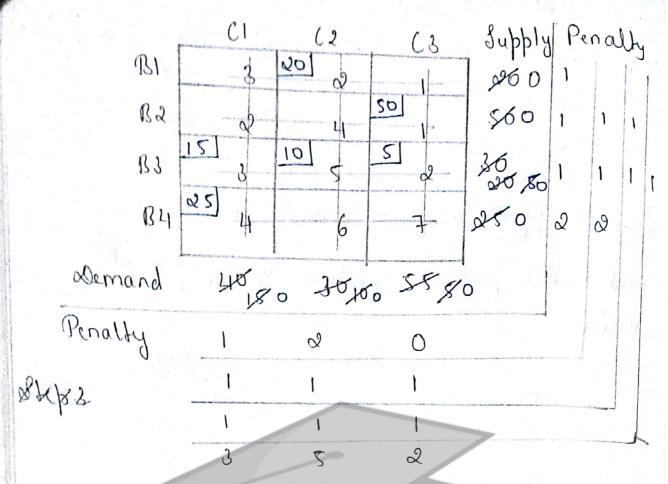
(2)		CI	C2	C3	Supply
	BI	5	7	8	70
	BJ	4	4	6	30
	BS	6	7	7	50
Dem	and	65	42	43	

=) Step 1: Supply = 70 + 30 + 50 = 150 Demand = 65+42+43 = 150

Supply - Demand . Hence the given transportation

problem is balanced.

Stips: Add a penalty whommore find a least will in the now and find the difference. The result is added to the penalty when.



Shep's find the maximum penalty of in both show and coloumn. Here the maximum penalty is a for both B4 and Ca. find the least cell in B4 and Ca and assign the cost. Here do is assigned to (1,0), B1 is completed.

Shep 4 (alculate new penalty for the remaining shows and column. Repeat step as so.

Repeat sheps a to 4 until all the shows and column are completed.

Modified Distribution Method.

Just

i) dolve the following transpositation problem by applying Vogus method and also check optimality

		82	८३	34	Supply
01	6		9	ડ	70
02	M.	5	્ર ∖	8	55
OI	10	12	4	7	90
Demand	85	35	50	45	

Step 1: Apply Vogus appeneximation method and find the total cost.

supply - Demand. Hence the given transportation peroblem is balanced.

J	31	S2	SI	34	Supply	Penal	. fy	
01	Comment of Comments	35	9 3	5]	70 2	3	3	
01	51	7.5	20	8	55 3	6	3	3
03	80	12	4	510	90 J	3	3 3	5
Dunand	85	20	\$60	450				
Penalty	4	Ц	હ	4				
3	4		હ	4				
	4			4		and the second		

Phase-II: MODI/OV, LOOP METHOD

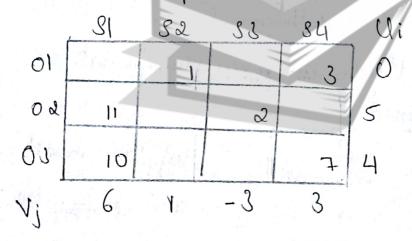
Check if the total numbers of allocations is

equal to m+n-1 m=no of slower, n=no of columns

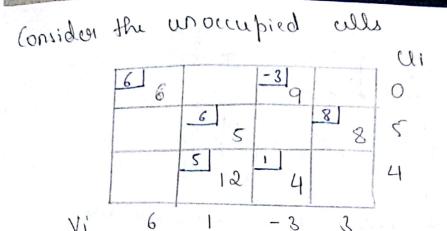
m+n-1 = total no of allocation

3+4-1 = 6 7-1 = 6 6 = 6

lonsides the occupied all

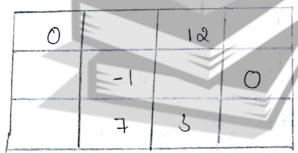


Calculate the values of Ui and Vi such that Ui+Vi = Cij. Start by initializing any one of the now on column value as O



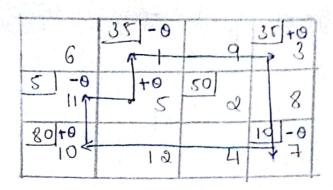
(alculate 2j for each unoccupied all such that 2j = Vj+Ui
(alculate (Cij-2j) for each ea all and heck

Calculate (Cij-2j) for each ea cell and order
if the condition Cij-2j>0. If the condition
is not statisfied than TC=1165 is not
obtimum solution.



Here the cell (2,2) how a negative value. Here the condition (ij-2j 20 is not satisfied.

Now consider the cell with the negative value ie (22) and form a closed loop to the op occupied cells and assign +1-0 to the alternate cells.



To calculate the value of O consider the all with negative theta values and find the minimum among them 0 = min(35-0, 5-0, 10-0) = 0

Pubshitute the theta values to the corresponding eally occupied alls and and calculate the total cost.

1	6	30	9	40 3
4	0 11	515	201	8
	10	12	4	5 7

$$T(=(30\times1)+(40\times3)+(0\times11)+(5\times5)+(50\times2)$$

$$=1160$$

Apply MODI IVV ON LOOP method again to check if the solution is optimum on not.

$$m+n-1$$
 = total no of allocations
 $3+4-1=6$

Consider the occupied cells

(alculate the Vellues of Ui and Vi such that

(li + Vi = Cij. Slant by initializing any one of the

slow on column value as 0

					Ui
		١		3	-4
	Promise and contract of conjunction appealance	5	ચ		0
	10			7	0
1 j	10	5	Q	7	

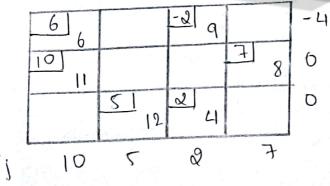
Consider the unoccupied cells.

Calculate 215 for each unoccupied cells such that

Calculate (ij - 21) for each cell

Zij = Vj+U1 and calculate (ij - 21) for each cell

and where if the condition (ij - 2j) = 20 is satisfied.

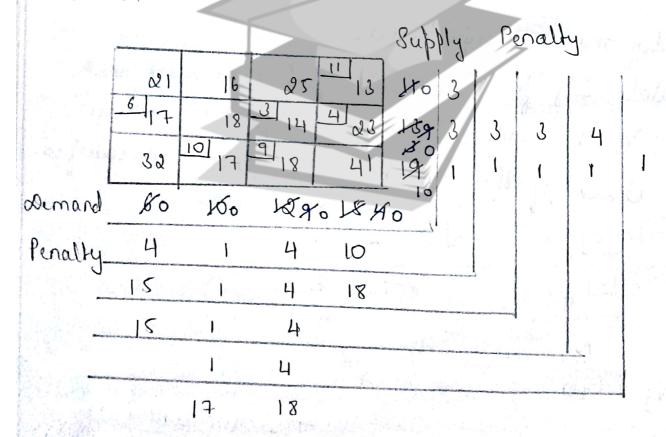


0	430	U	
1	and the contract of the contract of the		1
-		e presente sa secondoscente (THE MALINES HAVE ARREST
Stanton (7	2	

The condition is satisfied (cij-2j=0). Hence TC: 1160 is the optimum solution.

- Apply Vogus Approximation method and find the total cost

Supply-Demand. Hence the given peroblem is balanced.



11F c

Phase D: MODILOV, LOOP Method. that if the total number of allocations is equal to m+n-1 m=no of slows, n=no of columns m+n-1 = total no of allocations 3+4-1 = 6 6 > 6

Consider the occupied alls

		-		CONTRACTOR OF STREET	\mathcal{O}
	7			13	0
	17		14	23	10
The second second		17	18	43/	14
2	7	3	4	13	

(abulat the values of U and V; such that Ui+Vj = (ij . Stant by initializing any one of the now an wlump value by O

Consider the unoccupied alle

			The second secon	/ //
FU	3 16	4 25		0
	13			10
32			41	14
7	J	4	13	

Calculate 2; for each unoccupied all such that 2/2 /1+4;

(alculate (Cij - 2j) for each all and their if the condition (ij-2j20 Here the condition is satisfied and hence TC=711 is the optimal

rolution. Go paperless! Save Earth SOURCE: www.diginotes.in.

14	13	21	The second second second second second
	5	- 1 - 3	
11			14

Cij - 2 j 20

Hence TC=711 is the optimal solution.

Assignment Powblems:

Row operation: Find the minimum dement in each now and subtract it with other element

of the 9000. SI SZ SJ S4 ST A 8 1 1 0 6 7 B 7 5 6 0 5

c 5 3 4 0 2

D 1 3 6 0 &

Step 0: E L 3 4 3 04]

Column operation: find min in each column 4 subtract it with other element of the now.

 7
 0
 0
 4

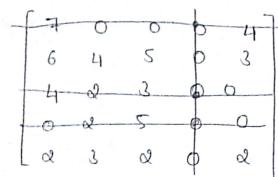
 6
 4
 5
 0
 3

 4
 2
 3
 0
 0

 0
 2
 5
 0
 0

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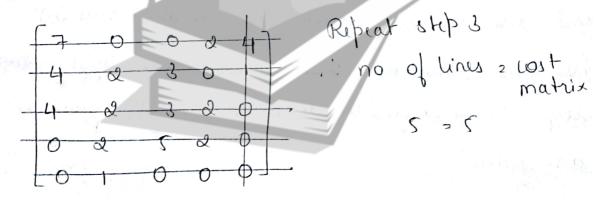
Duaw minimum hosizontal and vertical lines such that it should cover all the zero's.



check if no of lines = cost matrix. If equal jump to step 4 jump to step 5. If not equal jump to step 4 no of lines + cost matrix

4 = 5

ship 4 Consider unallocated elements and find the smallest cost. Substract remaining elements with the cost and add the cost to interection points



NOW RY

8ths

Consider the now on column with I zero and stuke out the other zero's of that the Go paperless! Save Earth SOURCE: www.diginotes.inned by CamScanner

allocated sow on column.

Tob Mlc

A -> 82 = 3

B -> 84 = 2

C -> 85 =
$$^{+}4$$

D -> 81 = $^{+}3$

E -> S3 = $^{+}9$

Maximization in assignment problem:

The objective is to maximize the perofit to solve this use first convoit the given perofit matrix into the closs matrix by substracting all the elements from the highest element. For this convoited closs matrix we apply the steps in hungarian method to get optimum assignment.

of: It marketing manager how 5 salesman and there are 5 districts considering the Capability of salesman and nature of districts. The estimates made by the marketing managers for the sales per month for each salesman in each district

could be as follows find the assignment of salisman to the districts that will rusult in the maximum sales

Step 1: Find the maximum element. Subtract all the elements of the matrix with the maximum. max - 40

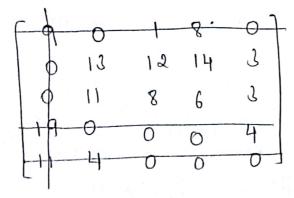
the now and subtract it with other element of the now and subtract it with other element of the now

step 3: Column operation: find the minimum element in each column and subtract it with other element of the column.

step 3: Deraw minimum hoodgontal and vertical lines such that it should cover all the zero's

their if no of lines = cost matrix. If equal jum
to step 5. If not equal superate step 24

Step 4: Consider un allocated eliments and find
the smallest cost. Subtract minimum eliment with
the smallest cost. Subtract to intersection points
the cost and add the cost to intersection points



no of lines of lost matrix
4 & 5
Repeat step 4

no of lines = cost matrix

5 = 5

step 5: Consider slow on column with one good and stake out the other zero's of the allocated slow on column.

