Module - 4

Transportation and Assignment Problems.

The transportation problem:

The transportation problem is to transport various amounts of a single homogeneous comodity, that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimum.

Methods to find initial feasible solution:

1. Northwest corner method (NWC)
2. Matrix Minima method
3. Vogels Approximation method. (VAM)

North West Corner Method (NWC)

Step 1: Identify the northwest corner of the table

Allocate $x_{11} = \min(a_1, b_1)$

case 1: If $a_1 < b_1$, then first row gets completed.

case 2: If $b_1 < a_1$, then first column gets completed

case 3: If $a_1 = b_1$, then there is a tie and allocation can be made arbitrarily.

Step 2: Start from the northwest corner and repeat step 1 until all the requirements are satisfied.

Q1: Find the initial feasible solution for the following transportation problem by ~~us~~ using north west corner method.

	C1	C2	C3	Supply
B1	3	2	1	20
B2	2	4	1	50
B3	3	5	2	30
B4	4	6	7	25
Demand	40	30	55	

Step 1: Supply = 20 + 50 + 30 + 25 = 125

Demand = 40 + 30 + 55 = 125

Supply = Demand, Hence the given transportation problem is balanced.

	C1	C2	C3	
B1	$\boxed{20}$ 3	2	1	~~20~~ 0
B2	$\boxed{20}$ 2	$\boxed{30}$ 4	1	~~50~~ ~~30~~ 0
B3	3	5	$\boxed{30}$ 2	~~30~~ 0
B4	4	6	$\boxed{25}$ 7	~~25~~ 0
	~~40~~ ~~20~~ 0	~~30~~ 0	~~55~~ ~~25~~ 0	

Step 2: The NWC is (1,1), $x_{11} = \min(20, 40) = 20$

B1 completes

20 is allocated to x_{11} (1,1)

Step 3: The NWC = (2, 1) $x_{21} = \min(20, 50) = 20$

20 is allocated to (2, 1), C1 completes.

Step 4: The NWC is (2, 2) $x_{22} = \min(30, 30) = 30$

30 is allocated to (2, 2) C2 and B2 are complete.

Step 5: The NWC is (3, 3) $x_{33} = \min(30, 55) = 30$

30 is allocated to (3, 3) B3 is complete.

Step 6: The NWC is (4, 3) $x_{43} = \min(25, 25) = 25$

25 is allocated to (4, 3) C3 and B4 are complete.

∴ The total cost

$TC = (20 \times 3) + (20 \times 2) + (30 \times 4) + (30 \times 2) + (25 \times 7)$

$= \underline{\underline{455}}$

Q2: Find initial feasible solution by applying northwest corner method.

	D_1	D_2	D_3	Supply
O1	5	7	8	70
O2	4	4	6	30
O3	6	7	7	50
demand	65	42	43	

Step 1: Supply = 70 + 30 + 50 = 150

Demand = 65 + 42 + 43 = 150

Supply = Demand, Hence given transportation problem is balanced.

	D_1	D_2	D_3	Supply
O_1	$\boxed{65}$ 5	$\boxed{5}$ 7	8	~~70~~ ~~5~~ 0
O_2	4	$\boxed{30}$ 4	6	~~30~~ 0
O_3	6	$\boxed{7}$ 7	$\boxed{43}$ 7	~~50~~ ~~43~~ 0
Demand	~~65~~ 0	~~42~~ ~~37~~ ~~7~~ 0	~~43~~ 0	

Step 2: The NWC is (1, 1), $x_{11} = \min(70, 65) = 65$

65 is allocated to (1, 1), D1 is complete

Step 3: The NWC is (1, 2) $x_{12} = \min(5, 42) = 5$

5 is allocated to (1, 2), O_1 is complete

Step 4: The NWC is (2, 2) $x_{22} = \min(30, 37) = 30$

30 is allocated to (2, 2) O_2 is complete

Step 5: The NWC is (3, 2) $x_{32} = \min(50, 7) = 7$

7 is allocated to (3, 2), D_2 is complete

Step 6: The NWC is (3, 3) $x_{33} = \min(43, 43) = 43$

43 is allocated to (3, 3) O_3 and D_3 are complete

∴ The total cost is

$TC = (65\times5) + (5\times7) + (30\times4) + (7\times7) + (43\times7)$

$= \underline{\underline{830}}$

Q 3). Find the feasible solution by applying northwest corner method.

	D_1	D_2	D_3	D_4	Supply
O_1	6	4	1	5	14
O_2	8	9	2	7	6
O_3	4	3	6	2	3
Demand	6	10	15	4	

Step 1 : Supply $= 14 + 6 + 3 = 23$

Demand $= 6 + 10 + 15 + 4 = 35$

Supply ≠ Demand. The problem is unbalanced.

Add a dummy row O_4 with cost 12 to $(35-23)$ to balance supply and demand.

	D_1	D_2	D_3	D_4	
O_1	$\boxed{6}$ 6	$\boxed{8}$ 4	1	5	$\cancel{14}$ $\cancel{8}$ 0
O_2	8	$\boxed{2}$ 9	$\boxed{4}$ 2	7	$\cancel{6}$ $\cancel{4}$ 0
O_3	4	3	$\boxed{3}$ 6	2	$\cancel{3}$ 0
O_4	0	0	$\boxed{8}$ 0	$\boxed{4}$ 0	$\cancel{12}$ $\cancel{4}$ 0
	$\cancel{6}$ 0	$\cancel{10}$ $\cancel{2}$ 0	$\cancel{15}$ $\cancel{11}$ $\cancel{8}$ 0	$\cancel{4}$ 0	

Step 2: The NWC is (1,1), $x_{11} = \min(14,6) = 6$
6 is allocated to (1,1), D_1 is complete

Step 3: The NWC is (1,2) $x_{12} = \min(8,10) = 8$
8 is allocated to (1,2) O_1 is complete

Step 4: The NWC is (2,2) $x_{22} = \min(2,6) = 2$
2 is allocated to (2,2) D_2 is complete

Step 5: The NWC is (2,3) $x_{23} = \min(15,4) = 4$
4 is allocated to (2,3) O_2 is complete

Step 6: The NWC is (3,3) $x_{33} = \min(3,11) = 3$
3 is allocated to (3,3) O_3 is complete

Step 7: The NWC is (4,3) $x_{43} = \min(8,12) = 8$
8 is allocated to (4,3) D_3 is complete

Step 8: The NWC is (4,4) $x_{44} = \min(4,4) = 4$
4 is allocated to (4,4) & O_4 and D_4 are complete.

∴ The total cost is

$TC = (6\times6) + (8\times4) + (2\times9) + (4\times2) + (6\times3) + (8\times0) + (4\times0)$

$= \underline{\underline{112}}$

Least cost method (Matrix Minima Method)

Step 1: Determine the smallest cost in the transporta-tion table. Let it be c_{ij}. Allocate $= \min(a_i, b_j)$

Step 2: i) If $x_{ij} = a_i$, then cross out ith row. Goto step 3.

ii) If $x_{ij} = b_j$, then cross out jth coloumn. Goto step 3

iii) If $x_{ij} = a_i = b_j$, then cross out ith row or jth coloumn, but not both.

Step 3: Repeat steps 1 and 2 for resulting transpor-tation table until all requirements are satisfied.

Step 4 Whenever minimum cost is not unique, make an arbitary choice among the minima.

Q1)

	D_1	D_2	D_3	Supply
S_1	3	2	1	20
S_2	2	4	1	50
S_3	3	5	2	30
S_4	4	6	7	25
Demand	40	30	55	

Step 1: Supply = 20 + 50 + 30 + 25 = 125

Demand = 40 + 30 + 55 = 125

Demand = Supply. Hence the given transportation problem is balanced.

	D_1	D_2	D_3	Supply
S_1	3	$\boxed{15}$ 2	$\boxed{5}$ 1	~~20~~ ~~15~~ 0
S_2	2	4	$\boxed{50}$ 1	~~50~~ 0
S_3	$\boxed{30}$ 3	5	2	~~30~~ 0
S_4	$\boxed{10}$ 4	$\boxed{15}$ 6	7	~~25~~ ~~15~~ 0
Demand	~~40~~ ~~10~~ 0	~~30~~ ~~15~~ 0	~~55~~ ~~50~~ 0	

Step 2: The least cost is 1, there is a tie between (1, 3) and (2, 3). Find out the cell to which maximum cost can be allocated i.e (2, 3) = 50, S2 is completed.

Step 3: The least cost is 1. Allocate / min

$\min_{13} = (5, 20) = 5$ Allocate 5 to (1, 3)

D3 is completed.

Step 4: The least cost is 2. x_{ij} $x_{12} = \min(30, 15) = 15$

Allocate 15 to (1, 2)

S1 is completed.

Step 5: The least cost is 3. $x_{31} = \min(30, 40) = 30$

Allocate 30 to (3, 1), S3 is complete

Step 6: The least cost is 4 $x_{41} = \min(10, 25) = 10$

Allocate 10 to (4,1) D_1 is complete.

Step 7: The least cost is 6 $x_{42} = \min(15, 15) = 15$

Allocate 15 to (4.2) D_2 is complete.

∴ Total cost

$TC = (15 \times 2) + (5 \times 1) + (50 \times 1) + (30 \times 3) + (10 \times 4)$

$+ (15 \times 6)$

$= \underline{\underline{305}}$

Q2)

	D1	D2	D3	Supply
O1	5	7	8	70
O2	4	4	6	30
O3	6	7	7	50
Demand	65	42	43	

Step 1: Supply $= 70 + 30 + 50 = 150$

Demand $= 65 + 42 + 43 = 150$

Demand = Supply. Hence the given transportation problem is balanced.

	D_1	D_2	D_3	Supply
O_1	$\boxed{35}$ 5	$\boxed{35}$ 7	8	~~70~~ ~~35~~ 0
O_2	$\boxed{30}$ 4	4	6	~~30~~ 0
O_3	6	$\boxed{7}$ 7	$\boxed{43}$ 7	~~50~~ ~~7~~ 0
Demand	~~65~~ ~~35~~ 0	~~42~~ ~~7~~ 0	~~43~~ 0	

Step 2: The least cost is 4. maximum cost can be allocated to (2,1) $x_{21} = \min(30, 65) = 30$

30 is allocated to (2,1) O_2 is completed.

Step 3: The least cost is 5. $x_{11} = \min(70, 35) = 35$

35 is allocated to (1,1) D_1 is completed.

Step 4: The least cost is 7, maximum cost can be allocated to (3,3) $x_{33} = \min(50, 43) = 43$

43 is allocated to (3,3) D_3 is completed.

Step 5: The least cost is 7. maximum cost can be allocated to (1,2) $x_{12} = \min(35, 42) = 35$

O_1 is completed

Step 6: The least cost is 7 ~~$x_{32} = (7,7)$~~

$x_{32} = \min(7, 7) = 7$ 7 is allocated to (3,2)

D_2 and O_3 are completed.

The total cost is :

$TC = (35 \times 5) + (35 \times 7) + (30 \times 4) + (7 \times 7) + (43 \times 7)$

$= \underline{890}$

Vogel's Approximation Method (VAM)

1. Find initial feasible solution by applying VAM method.

	C_1	C_2	C_3	Supply
B_1	3	2	1	20
B_2	2	4	1	50
B_3	3	5	2	30
B_4	4	6	7	25
Demand	40	30	55	

$\Rightarrow$ Supply $= 20 + 50 + 30 + 25 = 125$

Demand $= 40 + 30 + 55 = 125$

Supply = Demand. Hence the given transportation problem is balanced.

Step 2: Add a penalty column. Find 2 least cell in the row and find the difference. The result is added to the penalty coloumn.

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	C1	C2	C3	Supply	Penalty			
B1	3	$\boxed{20}$ 2	1	~~20~~ 0	1			
B2	2	4	$\boxed{50}$ 1	~~50~~ 0	1	1	1	
B3	$\boxed{15}$ 3	$\boxed{10}$ 5	$\boxed{5}$ 2	~~30~~ ~~20~~ ~~5~~ 0	1	1	1	1
B4	$\boxed{25}$ 4	6	7	~~25~~ 0	2	2		
Demand	~~40~~ ~~15~~ 0	~~30~~ ~~10~~ 0	~~55~~ ~~5~~ 0					
Penalty	1	2	0					
Step 2	1	1	1					
	1	1	1					
	3	5	2					

Step 3: Find the maximum penalty in both row and coloumn. Here the maximum penalty is 2 for both B4 and C2. Find the least cell in B4 and C2 and assign the cost.

Here 20 is assigned to (1, 2), B1 is completed.

Step 4: Calculate new penalty for the remaining rows and column. Repeat ~~step 2 to~~.

Repeat steps 2 to 4 until all the rows and column are completed.

Step 5: Calculate the total cost for all the allocated steps

Total cost:

$TC = (20\times2) + (50\times1) + (15\times3) + (10\times5) + (5\times2) + (25\times4)$

$= \underline{\underline{295}}$

Q 2)

	C1	C2	C3	Supply
B1	5	7	8	70
B2	4	4	6	30
B3	6	7	7	50
Demand	65	42	43	

⇒ Step 1: Supply = 70 + 30 + 50 = 150

Demand = 65 + 42 + 43 = 150

Supply = Demand. Hence the given transportation problem is balanced.

Step 2: Add a penalty column. Find 2 least cell in the row and find the difference. The result is added to the penalty column.

	C1	C2	C3	Supply	Penalty			
B1	3	$\boxed{20}$ 2	1	~~20~~ 0	1			
B2	2	4	$\boxed{50}$ 1	~~50~~ 0	1	1	1	
B3	$\boxed{15}$ 3	$\boxed{10}$ 5	$\boxed{5}$ 2	~~30~~ ~~20~~ ~~5~~0	1	1	1	1
B4	$\boxed{25}$ 4	6	7	~~25~~ 0	2	2		
Demand	~~40~~ ~~15~~ 0	~~30~~ ~~10~~ 0	~~55~~ ~~5~~ 0					
Penalty	1	2	0					
Step 2	1	1	1					
	1	1	1					
	3	5	2					

Step 3: Find the maximum penalty in both row and coloumn. Here the maximum penalty is 2 for both B4 and C2. Find the least cell in B4 and C2 and assign the cost. Here 20 is assigned to (1, 2), B1 is completed.

Step 4: Calculate new penalty for the remaining rows and column. Repeat ~~step 2 to~~

<u>Repeat steps 2 to 4 until all the rows and column are completed.</u>

Modified Distribution Method.

1) Solve the following transportation problem by applying Vogus method and also check optimality test

	S1	S2	S3	S4	Supply
O1	6	1	9	3	70
O2	11	5	2	8	55
O3	10	12	4	7	90
Demand	85	35	50	45	

Step 1:: Apply Vogus approximation method and find the total cost.

Supply = 70 + 55 + 90 = 215

Demand = 85 + 35 + 50 + 45 = 215

Supply = Demand. Hence the given transportation problem is balanced.

	S1	S2	S3	S4	Supply	Penalty			
O1	6	$\boxed{35}$ 1	9	$\boxed{35}$ 3	~~70~~ ~~35~~ 0	2	3	3	
O2	$\boxed{5}$ 11	5	$\boxed{50}$ 2	8	~~55~~ ~~50~~ 0	3	6	3	3
O3	$\boxed{80}$ 10	12	4	$\boxed{10}$ 7	~~90~~ ~~80~~ 0	3	3	3	3
Demand	~~85~~ 5	~~35~~ 0	~~50~~ 0	~~45~~ 0					
Penalty	4	4	2	4					
	4		2	4					
	4			4					
	1			1					

∴ The total cost is

$TC = 35 + (35 \times 3) + (5 \times 11) + (50 \times 2) + (10 \times 8) + (10 \times 7)$

$= 1165$

Phase - II : MODI / UV, LOOP METHOD

Check if the total numbers of allocations is equal to $m+n-1$ m = no of rows, n = no of column

$m+n-1$ = total no of allocation

$3+4-1 = 6$

$7-1 = 6$

$6 = 6$

Consider the occupied cell

	S1	S2	S3	S4	U_i
O1		1		3	0
O2	11		2		5
O3	10			7	4
V_j	6	1	-3	3	

Calculate the values of U_i and V_j such that $U_i + V_j = C_{ij}$. Start by initializing any one of the row or column value as 0

Consider the unoccupied cells

				u_i
$\boxed{6}$ 6		$\boxed{-3}$ 9		0
	$\boxed{6}$ 5		$\boxed{8}$ 8	5
	$\boxed{5}$ 12	$\boxed{1}$ 4		4
v_j: 6	1	-3	3	

Calculate z_j for each unoccupied cell such that $z_j = v_j + u_i$

Calculate $(c_{ij} - z_j)$ for each ~~ea~~ cell and check if the condition $c_{ij} - z_j \geq 0$. If the condition is not statisfied then $TC = 1165$ is not optimum solution.

0		12	
	-1		0
	7	3	

Here the cell (2,2) has a negative value. Hence the condition $c_{ij} - z_j \geq 0$ is not satisfied.

Now consider the cell with the negative value i.e (2,2) and form a closed loop to the of occupied cells and assign $+/-\theta$ to the alternate cells.

6	[35] −θ 1	9	[35] +θ 3
[5] −θ 11	+θ 5	[50] 2	8
[80] +θ 10	12	4	[10] −θ 7

To calculate the value of θ. Consider the cell with negative theta values and find the minimum among them

$\theta = \min(35-\theta,\ 5-\theta,\ 10-\theta) = 0$

$5-\theta = 0$

$\theta = 5$

Substitute the theta values to the corresponding cells occupied cells and and calculate the total cost.

6	[30] 1	9	[40] 3
[0] 11	[5] 5	[50] 2	8
[85] 10	12	4	[5] 7

$TC = (30\times1) + (40\times3) + (0\times11) + (5\times5) + (50\times2)$
$+ (85\times10) + (5\times7)$
$= 1160$

Apply MODI/UV or LOOP method again to check if the solution is optimum or not.

$m + n - 1 =$ total no of allocations

$3 + 4 - 1 = 6$

$6 = 6$

Consider the occupied cells

Calculate the values of U_i and V_j such that $U_i + V_j = C_{ij}$. Start by initializing any one of the row or column value as 0

				U_i
	1		3	-4
	5	2		0
10			7	0
V_j: 10	5	2	7	

Consider the unoccupied cells.

Calculate Z_{ij} for each unoccupied cells such that $Z_{ij} = V_j + U_i$ and calculate $(C_{ij} - Z_{ij})$ for each cell and check if the condition $(C_{ij} - Z_{ij}) \geq 0$ is satisfied.

				U_i
$\boxed{6}$ 6		$\boxed{-2}$ 9		-4
$\boxed{10}$ 11			$\boxed{7}$ 8	0
	$\boxed{5}$ 12	$\boxed{2}$ 4		0
V_j: 10	5	2	7	

0		11	
1			1
	7	2	

The condition is satisfied $(C_{ij} - Z_{j} \geq 0)$

Hence $TC = 1160$ is the optimum solution.

2)

	D1	D2	D3	D4	Supply
S1	21	16	25	13	11
S2	17	18	14	23	13
S3	32	17	18	41	19
Demand	6	10	12	15	

⟹ Apply Vogus Approximation method and find the total cost

Supply = 11 + 13 + 19 = 43

Demand = 6 + 10 + 12 + 15 = 43

Supply = Demand. Hence the given problem is balanced.

					Supply	Penalty				
	21	16	25	[11] 13	~~11~~ 0	3				
	[6] 17	18	[3] 14	[4] 23	~~13~~ ~~9~~ ~~3~~ 0	3	3	3	4	
	32	[10] 17	[9] 18	41	~~19~~ ~~9~~ 10	1	1	1	1	1
Demand	~~6~~ 0	~~10~~ 0	~~12~~ ~~9~~ 0	~~15~~ ~~4~~ 0						
Penalty	4	1	4	10						
	15	1	4	18						
	15	1	4							
		1	4							
		17	18							

TC = (11 × 13) + (6 × 17) + (3 × 14) + (4 × 23) + (10 × 17) + (9 × 18)

= 711

Phase II : MODI / UV, LOOP Method.

Check if the total number of allocations is equal to $m+n-1$ m = no of rows, n = no of columns

$m+n-1$ = total no of allocations

$3+4-1 = 6$

$6 = 6$

Consider the occupied cells

				U_i
			13	0
17		14	23	10
	17	18		14
7	3	4	13	V_j

Calculate the values of U_i and V_j such that $U_i + V_j = C_{ij}$. Start by initializing any one of the row or column value by 0

Consider the unoccupied cells

				U_i
$\boxed{7}$ 21	$\boxed{3}$ 16	$\boxed{4}$ 25		0
	$\boxed{13}$ 18			10
$\boxed{21}$ 32			$\boxed{27}$ 41	14
7	3	4	13	V_j

Calculate Z_j for each unoccupied cell such that $Z_j = V_j + U_i$

Calculate $(C_{ij} - Z_j)$ for each cell and check if the condition $C_{ij} - Z_j \geq 0$ Here the condition is satisfied and hence TC = 711 is the optimal solution.

14	13	21	
	5		
11			14

$C_{ij} - Z_j \geq 0$

Hence TC = 711 is the optimal solution.

Assignment Problems:

i)

$$\begin{array}{c|ccccc} & S1 & S2 & S3 & S4 & S5 \\ \hline A & 10 & 3 & 3 & 2 & 8 \\ B & 9 & 7 & 8 & 2 & 7 \\ C & 7 & 5 & 6 & 2 & 4 \\ D & 3 & 5 & 2 & 2 & 4 \\ E & 9 & 10 & 9 & 6 & 10 \end{array}$$

⇒ Step 1.

Row operation: Find the minimum element in each row and subtract it with other element of the row.

$$\begin{array}{c|ccccc} & S1 & S2 & S3 & S4 & S5 \\ \hline A & 8 & 1 & 1 & 0 & 6 \\ B & 7 & 5 & 6 & 0 & 5 \\ C & 5 & 3 & 4 & 0 & 2 \\ D & 1 & 3 & 6 & 0 & 2 \\ E & 3 & 4 & 3 & 0 & 4 \end{array}$$

Step 2:

Column operation: find min in each column & subtract it with other element of the row.

$$\begin{bmatrix} 7 & 0 & 0 & 0 & 4 \\ 6 & 4 & 5 & 0 & 3 \\ 4 & 2 & 3 & 0 & 0 \\ 0 & 2 & 5 & 0 & 0 \\ 2 & 3 & 2 & 0 & 2 \end{bmatrix}$$

step 3:
Draw minimum horizontal and vertical lines such that it should cover all the zero's.

$$\begin{bmatrix} 7 & 0 & 0 & 0 & 4 \\ 6 & 4 & 5 & 0 & 3 \\ 4 & 2 & 3 & 0 & 0 \\ 0 & 2 & 5 & 0 & 0 \\ 2 & 3 & 2 & 0 & 2 \end{bmatrix}$$

check if no of lines = cost matrix. If equal jump to step 5. If not equal jump to step 4

no of lines ≠ cost matrix
4 ≠ 5

step 4: Consider unallocated elements and find the smallest cost. Substract remaining elements with the cost and add the cost to intersection points

$$\begin{bmatrix} 7 & 0 & 0 & 2 & 4 \\ 4 & 2 & 3 & 0 & 1 \\ 4 & 2 & 3 & 2 & 0 \\ 0 & 2 & 5 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Repeat step 3
∴ no of lines = cost matrix
5 = 5

Now R1

step 5:

$$\begin{bmatrix} 7 & \boxed{0} & \cancel{0} & 2 & 4 \\ 4 & 2 & 3 & \boxed{0} & 1 \\ 4 & 2 & 3 & 2 & \boxed{0} \\ \boxed{0} & 2 & 5 & 2 & \cancel{0} \\ \cancel{0} & 1 & \boxed{0} & \cancel{0} & \cancel{0} \end{bmatrix}$$

Consider the row or column with 1 zero and strike out the other zero's of that the allocated row or column.

Job		M/c		
A	→	S2	=	3
B	→	S4	=	+ 2
C	→	S5	=	+ 4
D	→	S1	=	+ 3
E	→	S3	=	+ 9
				21

Maximization in assignment problem:

The objective is to maximize the profit to solve this we first convert the given profit matrix into the loss matrix by substracting all the elements from the highest element. For this converted loss matrix we apply the steps in hungarian method to get optimum assignment.

Q: A marketing manager has 5 salesman and there are 5 districts considering the capability of salesman and nature of districts. The estimates made by the marketing managers for the sales per month for each salesman in each district could be as follows find the assignment of salesman to the districts that will result in the maximum sales

$$\begin{bmatrix} 32 & 38 & 40 & 28 & 40 \\ 40 & 24 & 28 & 21 & 36 \\ 41 & 27 & 33 & 30 & 37 \\ 22 & 38 & 41 & 36 & 36 \\ 29 & 33 & 40 & 35 & 39 \end{bmatrix}.$$

Step 1: Find the maximum element. Subtract all the elements of the matrix with the maximum.

max = 40

$$\begin{bmatrix} 9 & 3 & 1 & 13 & 1 \\ 1 & 17 & 13 & 20 & 5 \\ 0 & 14 & 8 & 11 & 4 \\ 19 & 3 & 0 & 5 & 5 \\ 12 & 8 & 1 & 6 & 2 \end{bmatrix}$$

Step 2: Row operation : Find the minimum element in each row and subtract it with other element of the row

min = 1

$$\begin{bmatrix} 8 & 2 & 0 & 12 & 0 \\ 0 & 16 & 12 & 19 & 4 \\ 0 & 14 & 8 & 11 & 4 \\ 19 & 3 & 0 & 5 & 5 \\ 11 & 7 & 0 & 5 & 1 \end{bmatrix}$$

Step 3: Column operation : Find the minimum element in each column and subtract it with other element of the column

$$\begin{bmatrix} 8 & 0 & 0 & 7 & 0 \\ 0 & 14 & 12 & 14 & 4 \\ 0 & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & 0 & 5 \\ 11 & 5 & 0 & 0 & 1 \end{bmatrix}$$

Step 3: Draw minimum horizontal and vertical lines such that it should cover all the zero's

$$\begin{bmatrix} 8 & 0 & 0 & 7 & 0 \\ 0 & 14 & 12 & 14 & 4 \\ 0 & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & 0 & 5 \\ 11 & 5 & 0 & 0 & 1 \end{bmatrix}$$

Check if no of lines = cost matrix. If equal jum to step 5. If not equal repeat step 4

Step 4: Consider un allocated elements and find the smallest cost. Subtract minimum element with the cost and add the cost to intersection poinb

$$\begin{bmatrix} 9 & 0 & 1 & 8 & 0 \\ 0 & 13 & 12 & 14 & 3 \\ 0 & 11 & 8 & 6 & 3 \\ 19 & 0 & 0 & 0 & 4 \\ 11 & 4 & 0 & 0 & 0 \end{bmatrix}$$

no of lines $\neq$ cost matrix

4 $\neq$ 5

Repeat step 4

$$\begin{bmatrix} 12 & 0 & 1 & 8 & 0 \\ 0 & 10 & 9 & 11 & 0 \\ 0 & 8 & 5 & 3 & 0 \\ \text{[illegible]} & 0 & 0 & 0 & 4 \\ 14 & 4 & 0 & 0 & 0 \end{bmatrix}$$

no of lines $\neq$ 0

no of lines = cost matrix

5 = 5

Step 5: Consider row or column with one zero and strike out the other zero's of the allocated row or column.

$$\begin{bmatrix} 12 & \boxed{0} & 1 & 8 & \cancel{0} \\ \boxed{0} & 10 & 9 & 11 & \cancel{0} \\ \cancel{0} & 8 & 5 & 3 & \boxed{0} \\ 3 & \cancel{0} & \boxed{0} & \cancel{0} & 4 \\ 14 & 4 & \cancel{0} & \boxed{0} & \cancel{0} \end{bmatrix}$$

$TC = 38 + 40 + 37 + 41 + 35 = \underline{\underline{191}}$